Exercise 66

Find the nth derivative of each function by calculating the first few derivatives and observing the pattern that occurs.

(a) $f(x) = x^n$ (b) $f(x) = 1/x$

Solution

Part (a)

Use the power rule to differentiate the function.

$$\begin{aligned} f'(x) &= nx^{n-1} \\ f''(x) &= n(n-1)x^{n-2} \\ f'''(x) &= n(n-1)(n-2)x^{n-3} \end{aligned}$$

Recognizing the pattern, the nth derivative is

$$f^{(n)}(x) = n!x^{n-n} = n!.$$

Part (b)

Rewrite the given function.

$$f(x) = x^{-1}$$

Use the power rule to differentiate the function.

$$\begin{aligned} f'(x) &= -x^{-2} \\ f''(x) &= -(-2)x^{-3} \\ f'''(x) &= -(-2)(-3)x^{-4} \end{aligned}$$

Recognizing the pattern, the nth derivative is

$$f^{(n)}(x) = (-1)^n n!x^{-1-n}.$$